## Exercise 2.3

**1** Identify the like terms in each set.

**a** $6x, -2y, 4x, x$ **b** $x, -3y, \frac{3}{4}, -5y$ **c** $ab, 4b, -4ba, 6a$

**d** $2, -2x, 3xy, 3x, -2y$ **e** $5a, 5ab, ab, 6a, 5$ **f** $-1xy, -yx, -2y, 3, 3x$

**2** Simplify by adding or subtracting like terms.

**a** $2y + 6y$ **b** $9x - 2x$ **c** $10x + 3x$

**d** $21x + x$ **e** $7x - 2x$ **f** $4y - 4y$

**g** $9x - 10x$ **h** $y - 4y$ **i** $5x - x$

**j** $9xy - 2xy$ **k** $6pq - 2qp$ **l** $14xyz - xyz$

**m** $4x^2 - 2x^2$ **n** $9y^2 - 4y^2$ **o** $y^2 - 2y^2$

**p** $14ab^2 - 2ab^2$ **q** $9x^2y - 4x^2y$ **r** $10xy^2 - 8xy^2$

**3** Simplify.

**a** $2x + y + 3x$ **b** $4y - 2y + 4x$ **c** $6x - 4x + 5x$

**d** $10 + 4x - 6$ **e** $4xy - 2y + 2xy$ **f** $5x^2 - 6x^2 + 2x$

**g** $5x + 4y - 6x$ **h** $3y + 4x - x$ **i** $4x + 6y + 4x$

**j** $9x - 2y - x$ **k** $12x^2 - 4x + 2x^2$ **l** $12x^2 - 4x^2 + 2x^2$

**m** $5xy - 2x + 7xy$ **n** $xy - 2xz + 7xy$ **o** $3x^2 - 2y^2 - 4x^2$

**p** $5x^2y + 3x^2y - 2xy$ **q** $4xy - x + 2yx$ **r** $5xy - 2 + xy$

**MATHEMATICAL CONNECTIONS**

You will need to be very comfortable with simplifying algebraic expressions. It is a skill you will need throughout the course for solving equations and inequalities, and for simplifying expansions.

**4** Simplify as far as possible.

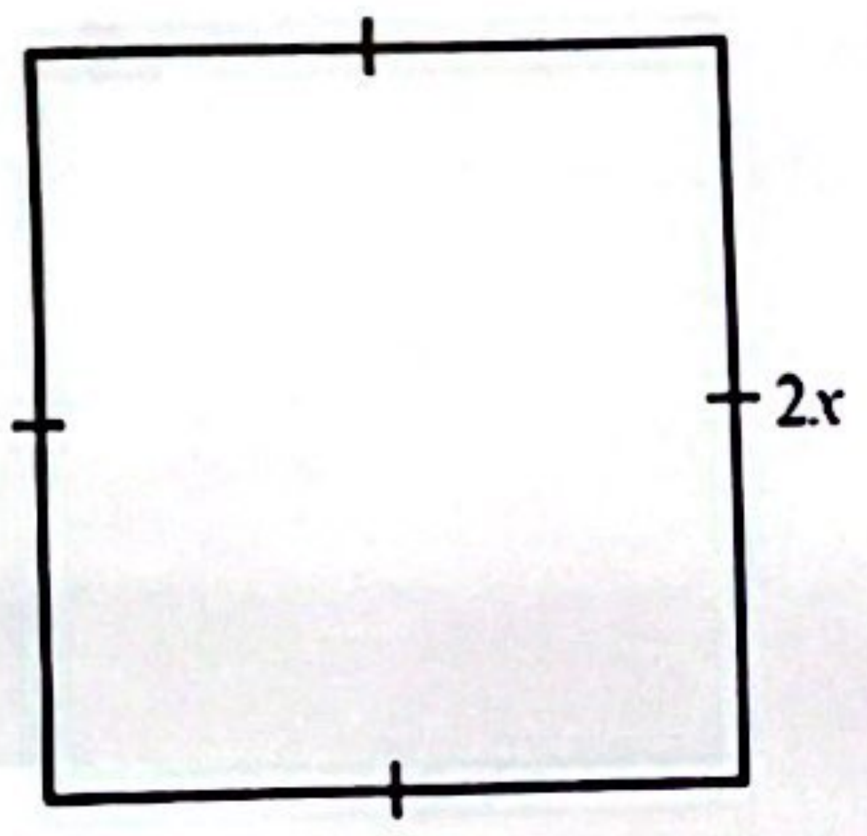
**a** $8y - 4 - 6y - 4$ **b** $x^2 - 4x + 3x^2 - x$ **c** $5x + y + 2x + 3y$

**d** $y^2 + 2y + 3y - 7$ **e** $x^2 - 4x - x + 3$ **f** $x^2 + 3x - 7 + 2x$

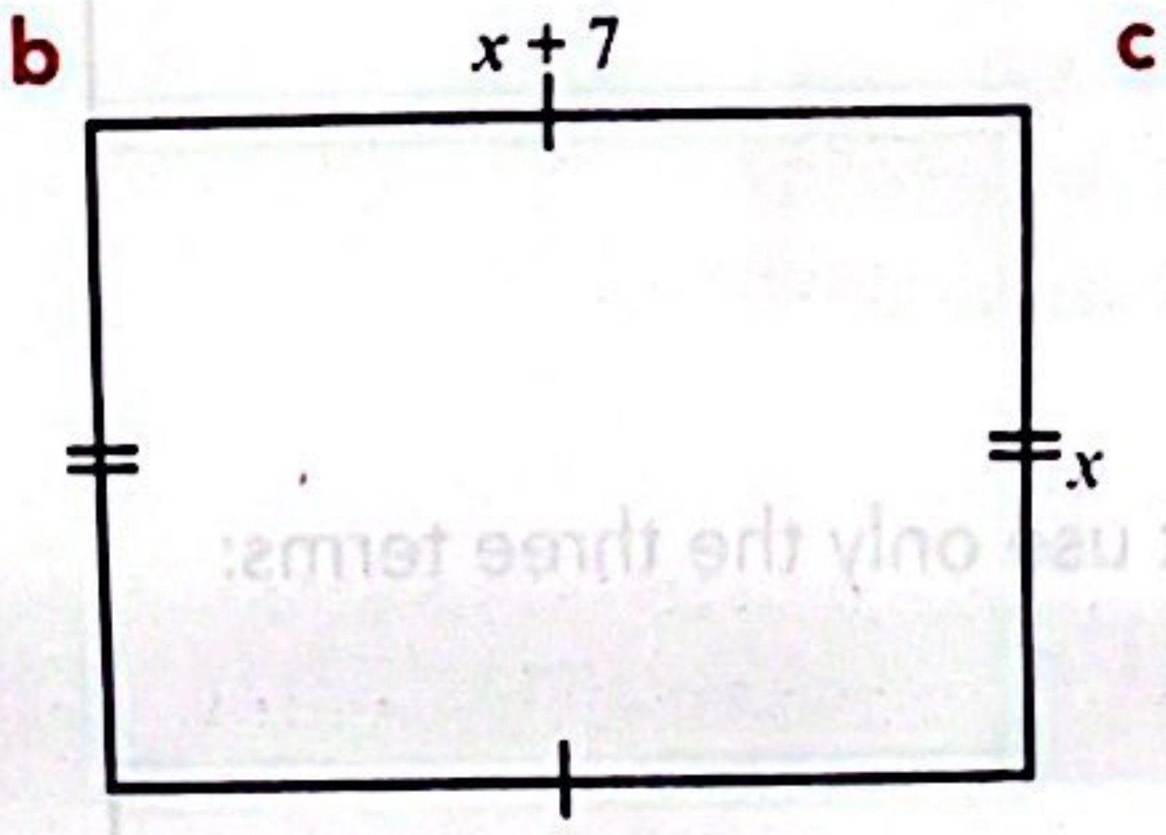
**g** $4xyz - 3xy + 2xz - xyz$ **h** $5xy - 4 + 3yx - 6$ **i** $8x - 4 - 2x - 3x^2$

**5** Write an expression for the perimeter ($P$) of each of the following shapes and then simplify it to give $P$ in the simplest possible terms.

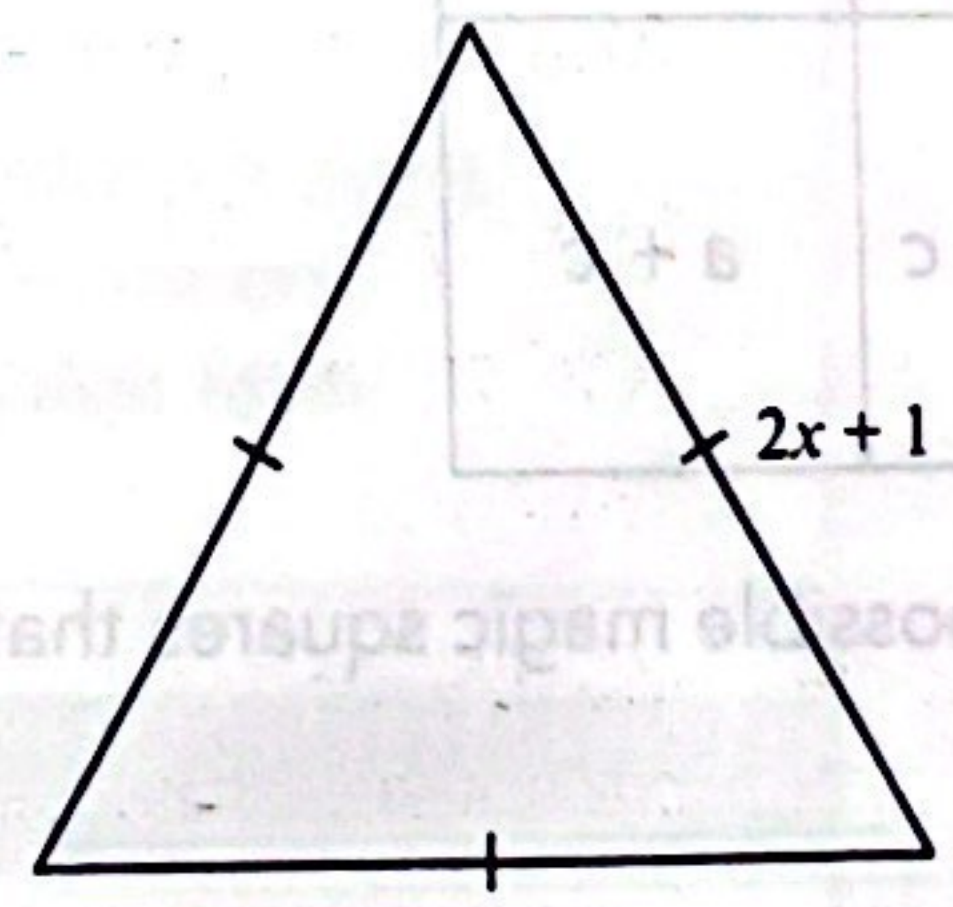
**a**



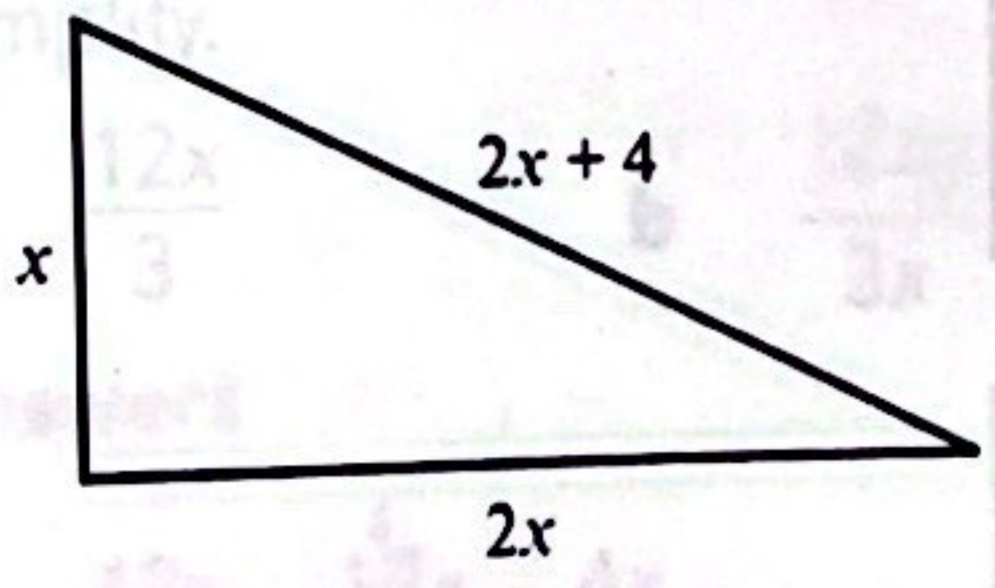
**b**



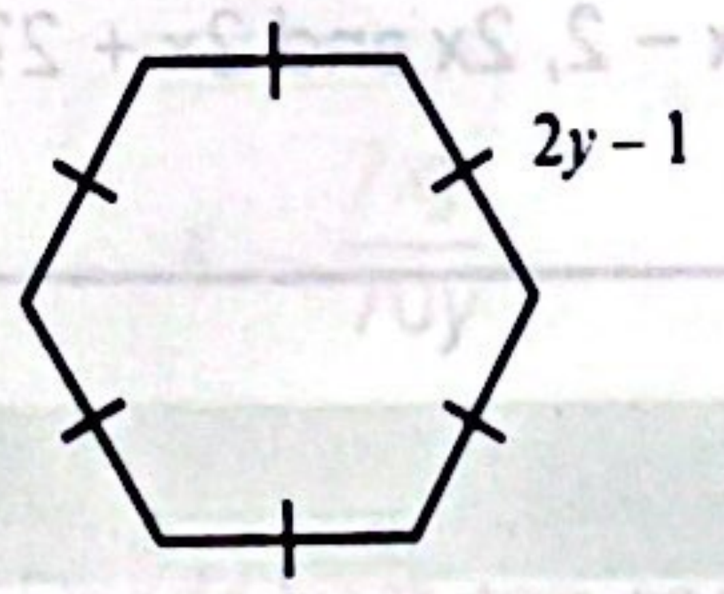
**c**



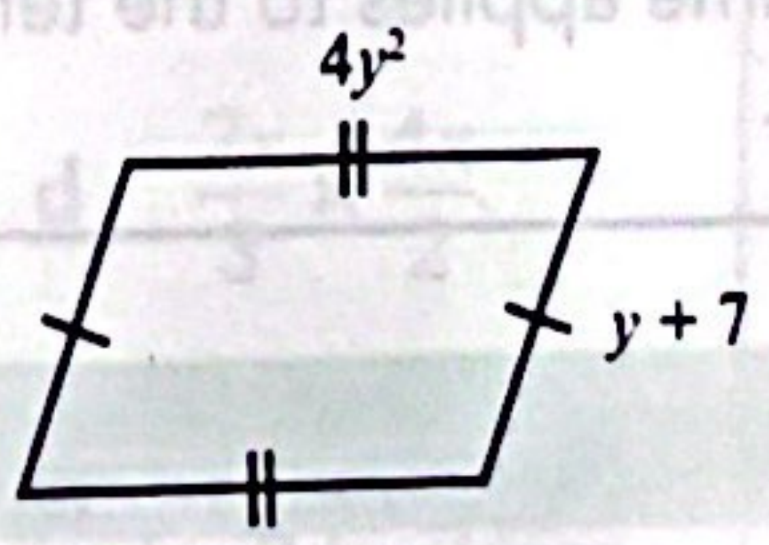
**d**



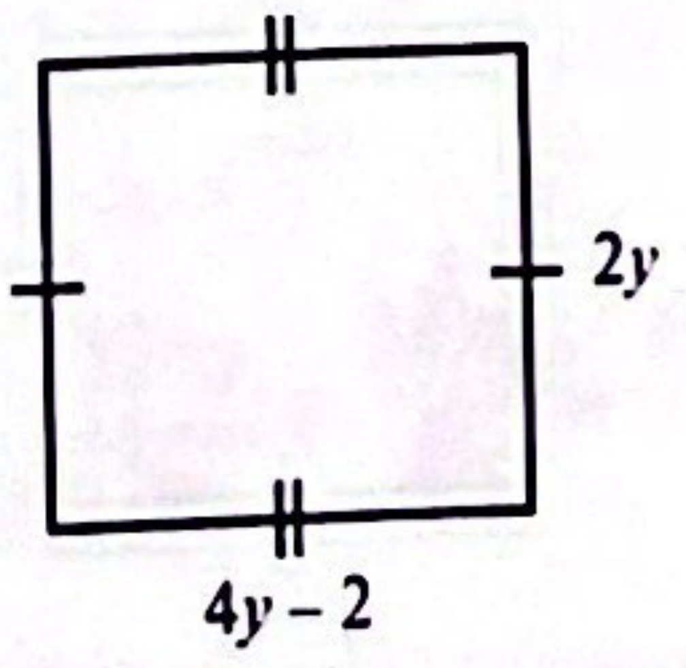
**e**



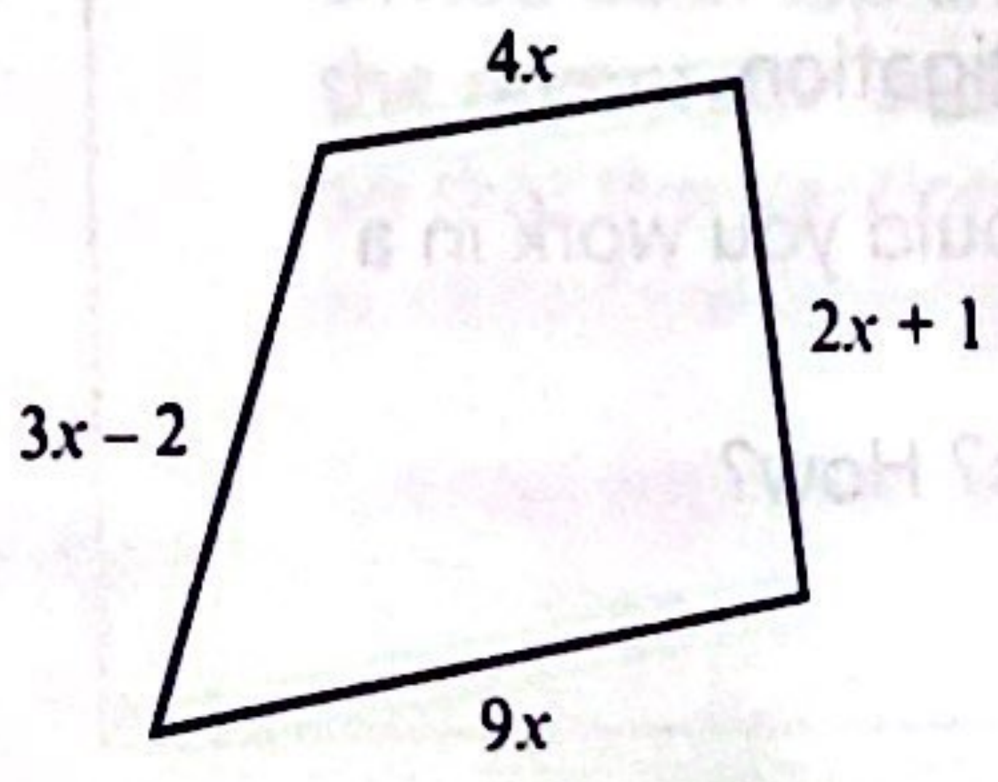
**f**



**g**



**h**



## WORKED EXAMPLE 8 CONTINUED

**c** $\frac{7xy}{70y} = \frac{\overset{1}{\cancel{7}}x\cancel{y}}{\underset{10}{\cancel{70}}\cancel{y}} = \frac{x}{10}$ Cancel.

**d** $\frac{2x}{3} \times \frac{4x}{2} = \frac{2 \times x \times 4 \times x}{3 \times 2}$ Insert × signs and multiply.

$= \frac{{}^{4}\cancel{8}x^2}{\cancel{6}_3}$ Cancel.

$= \frac{4x^2}{3}$

or

$\frac{{}^{1}\cancel{2}x}{3} \times \frac{4x}{\cancel{2}_1} = \frac{1x}{3} \times \frac{4x}{1} = \frac{4x^2}{3}$ Cancel first, then multiply.

## Exercise 2.4

**1** Multiply.

**a** $2 \times 6x$
**b** $4y \times 2$
**c** $3m \times 4$
**d** $2x \times 3y$
**e** $4x \times 2y$
**f** $9x \times 3y$
**g** $8y \times 3z$
**h** $2x \times 3y \times 2$
**i** $4xy \times 2xy$
**j** $4xy \times 2x$
**k** $9y \times 3xy$
**l** $4y \times 2x \times 3y$
**m** $2a \times 4ab$
**n** $3ab \times 4bc$
**o** $6abc \times 2a$
**p** $8abc \times 2ab$
**q** $4 \times 2ab \times 3c$
**r** $12x^2 \times 2 \times 3y^2$

**2** Simplify.

**a** $3 \times 2x \times 4$
**b** $5x \times 2x \times 3y$
**c** $2x \times 3y \times 2xy$
**d** $xy \times xz \times x$
**e** $2 \times 2 \times 3x \times 4$
**f** $4 \times 2x \times 3x^2y$
**g** $x \times y^2 \times 4x$
**h** $2a \times 3ab \times 2c$
**i** $10x \times 2y \times 3$
**j** $4 \times x \times 2 \times y$
**k** $9 \times x^2 \times xy$
**l** $4xy^2 \times 2x^2y$
**m** $7xy \times 2xz \times 3yz$
**n** $4xy \times 2x^2y \times 7$
**o** $9 \times xyz \times 4xy$
**p** $3x^2y \times 2xy^2 \times 3xy$
**q** $9x \times 2xy \times 3x^2$
**r** $2x \times xy^2 \times 3xy$

**3** Simplify.

**a** $\frac{15r}{3}$
**b** $\frac{40r}{10}$
**c** $\frac{21r}{7}$
**d** $\frac{12rs}{2r}$
**e** $\frac{14rs}{2s}$
**f** $\frac{18r^2s}{9r^2}$
**g** $\frac{10rs}{40r}$
**h** $\frac{15r}{60rs}$
**i** $\frac{7rst}{14rs}$
**j** $\frac{6rs}{r}$
**k** $\frac{r}{4r}$
**l** $\frac{r}{9r}$

**4** Simplify.

**a** $8x \div 2$ **b** $12xy \div 2x$ **c** $16x^2 \div 4xy$ **d** $24xy \div 3xy$

**e** $14x^2 \div 2y^2$ **f** $24xy \div 8y$ **g** $8xy \div 24y$ **h** $9x \div 36xy$

**i** $\frac{77xyz}{11xz}$ **j** $\frac{45xy}{20x}$ **k** $\frac{60x^2y^2}{15xy}$ **l** $\frac{100xy}{25x^2}$

**5** Simplify these as far as possible.

**a** $\frac{a}{2} \times \frac{b}{3}$ **b** $\frac{a}{3} \times \frac{a}{4}$ **c** $\frac{ab}{2} \times \frac{5a}{3}$ **d** $\frac{2a}{3} \times \frac{5}{b}$

**e** $\frac{2a}{4} \times \frac{3b}{4}$ **f** $\frac{5a}{2} \times \frac{5a}{2}$ **g** $\frac{a}{b} \times \frac{2b}{a}$ **h** $\frac{ab}{3} \times \frac{a}{b}$

**i** $5b \times \frac{2a}{5}$ **j** $4 \times \frac{2a}{3}$ **k** $\frac{a}{6} \times \frac{3}{2a}$ **l** $\frac{5a}{2} \times \frac{4a}{10}$

## 2.4 Working with brackets

When an expression has brackets, you normally have to remove the brackets before you can simplify the expression. Removing the brackets is called expanding the expression.

To remove brackets you multiply each term inside the bracket by the number (and/or variables) outside the bracket. When you do this you need to pay attention to the positive and negative signs in front of the terms:

$x(y + z) = xy + xz$ $x(y - z) = xy - xz$

$-x(y + z) = -xy - xz$ $-x(y - z) = -xy + xz$

**MATHEMATICAL CONNECTIONS**

You will learn more about expanding expressions to remove brackets in Chapter 10.

**TIP**

Expanding brackets is really just multiplying, so the same rules you used for multiplication apply in these examples.

**WORKED EXAMPLE 9**

Expand the following expressions.

**a** $2(2x + 6)$ **b** $4(7 - 2x)$ **c** $2x(x + 3y)$ **d** $xy(2 - 3x)$

**Answers**

**a**

$$2(2x + 6) = \overset{\text{i}}{2 \times 2x} + \overset{\text{ii}}{2 \times 6}$$
$$= 4x + 12$$

**b**

$$4(7 - 2x) = \overset{\text{i}}{4 \times 7} - \overset{\text{ii}}{4 \times 2x}$$
$$= 28 - 8x$$

For parts (a) to (d) write out the expression, or do the multiplication mentally.

Follow these steps when multiplying by a term outside a bracket:

- Multiply the term on the left-hand inside of the bracket first – shown by the red arrow labelled i.
- Then multiply the term on the right-hand side – shown by the blue arrow labelled ii.
- Then add the answers together.

### WORKED EXAMPLE 9 CONTINUED

**c**

$$\overset{\overset{\text{i}}{\curvearrowright}}{2x}(x+3y) = \overset{\text{i}}{2x \times x} + \overset{\text{ii}}{2x \times 3y}$$

$$\underset{\text{ii}}{\smile}$$

$$= 2x^2 + 6xy$$

**d**

$$\overset{\overset{\text{i}}{\curvearrowright}}{xy}(2-3x) = \overset{\text{i}}{xy \times 2} - \overset{\text{ii}}{xy \times 3x}$$

$$\underset{\text{ii}}{\smile}$$

$$= 2xy - 3x^2y$$

## Exercise 2.5

**1** **Expand:**

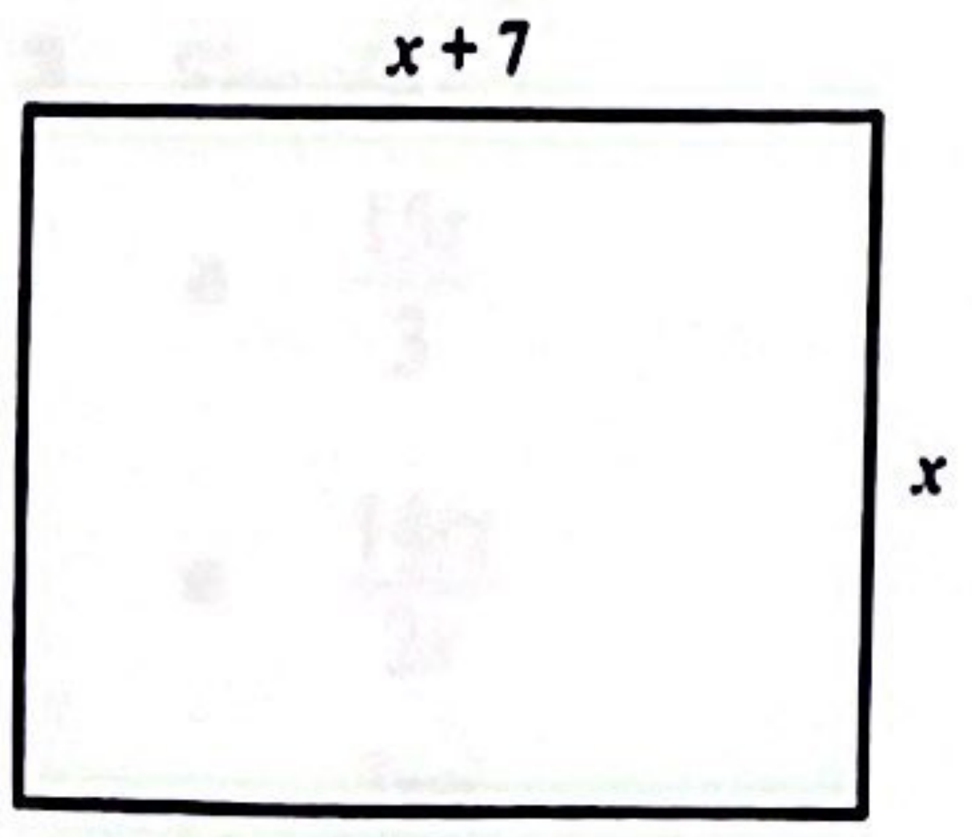
a $2(x+6)$
b $3(x+2)$
c $4(2x+3)$
d $10(x-6)$
e $4(x-2)$
f $3(2x-3)$
g $5(a+4)$
h $6(4+a)$
i $9(a+2)$
j $7(2c-2d)$
k $2(3c-2d)$
l $4(c+4d)$
m $5(2x-2y)$
n $6(3x-2y)$
o $3(4y-2x)$
p $4(s-4t^2)$
q $9(t^2-s)$
r $7(4t+t^2)$

**2** Remove the brackets to expand these expressions.

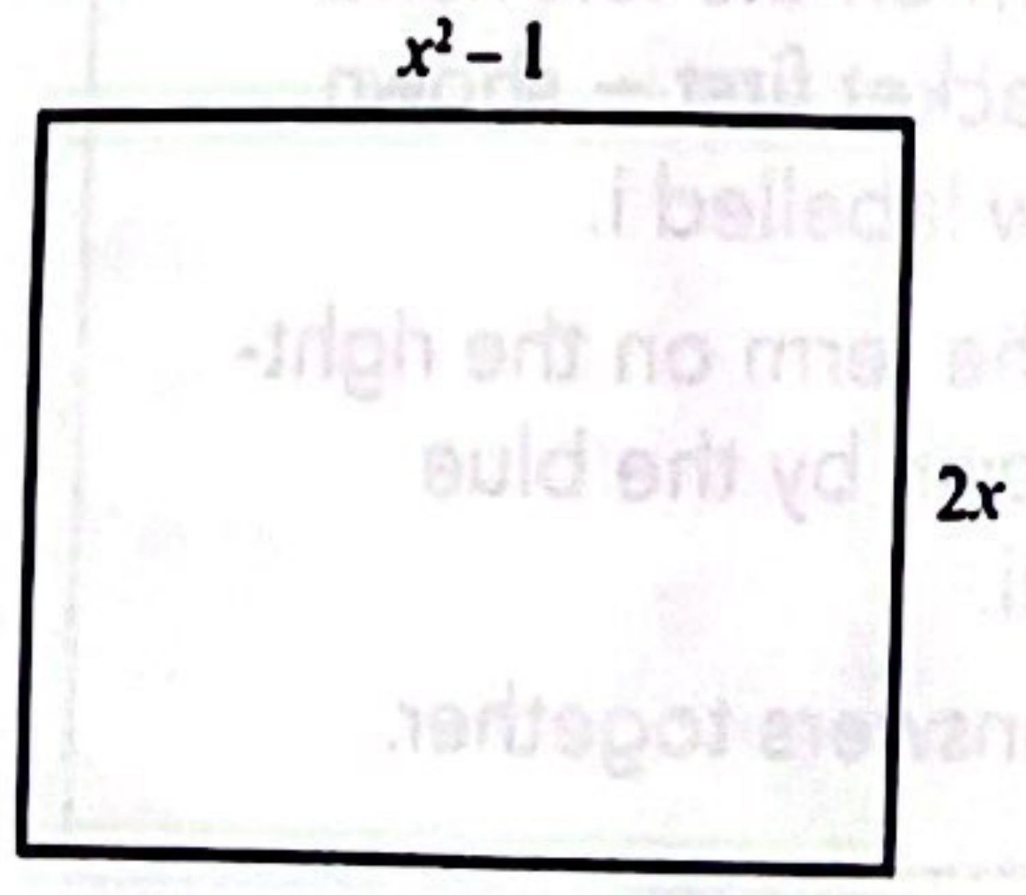
a $2x(x+y)$
b $3y(x-y)$
c $2x(x+2y)$
d $4x(3x-2y)$
e $xy(x-y)$
f $3y(4x+2)$
g $2ab(9-4b)$
h $2a^2(3-2b)$
i $3a^2(4-4b)$
j $4a(9-2b)$
k $5b(2-a)$
l $3a(4-b)$
m $2x^2y(y-2x)$
n $4xy^2(3-2x)$
o $3xy^2(x+y)$
p $x^2y(2x+y)$
q $9x^2(9-2x)$
r $4xy^2(3-x)$

**3** Given the formula for area, $A$ = length × width, write an expression for $A$ in terms of $x$ for each of the following rectangles. Expand the expression to give $A$ in simplest terms.

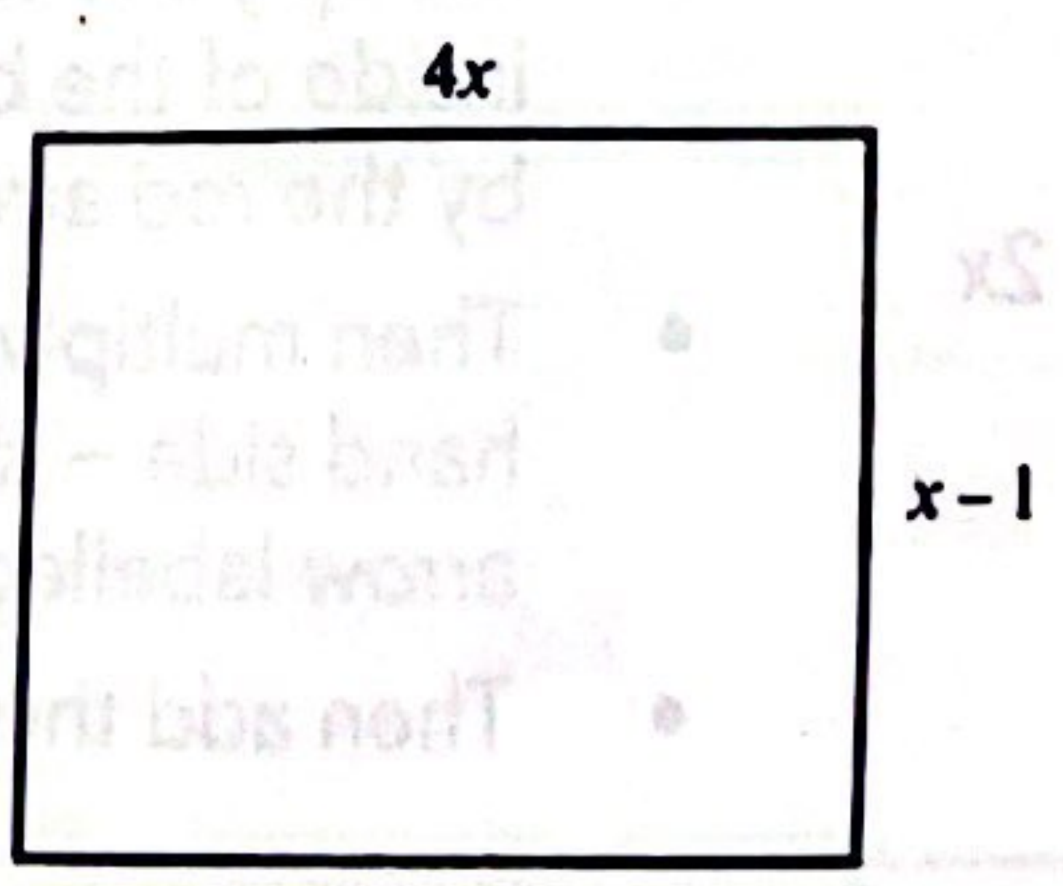
a



b



c



## Exercise 2.6

1 Expand and simplify.

- **a** $2(5 + x) + 3x$
- **b** $3(y - 2) + 4y$
- **c** $2x + 2(x - 4)$
- **d** $4x + 2(x - 3)$
- **e** $2t(4 + t) - 5$
- **f** $4(x + 2) - 7$
- **g** $6 + 3(x - 2)$
- **h** $4x + 2(2x + 3)$
- **i** $2x + 3 + 2(2x + 3)$
- **j** $3(2h + 2) - 3h - 4$
- **k** $6d + 2(d + 3)$
- **l** $7y + y(x - 4) - 4$
- **m** $2x(x + 4) - 4$
- **n** $2y(2x - 2y + 4)$
- **o** $2s(5 - 4s) - 4s^2$
- **p** $3x(2x + 4) - 9$
- **q** $3y(y + 2) - 4y^2$
- **r** $2(x - 1) + 4x - 4$

2 Simplify these expressions by removing brackets and collecting like terms.

- **a** $4(x + 40) + 2(x - 3)$
- **b** $2(x - 2) + 2(x + 3)$
- **c** $3(x + 2) + 4(x + 5)$
- **d** $8(x + 10) + 4(3 - 2x)$
- **e** $4(x^2 + 2) + 2(4 - x^2)$
- **f** $4p(p + 1) + 2p(p + 3)$
- **g** $3p(4q - 4) + 4(3pq + 4p)$
- **h** $2x(5y - 4) + 2(6x - 4xy)$
- **i** $3x(4 - 8y) + 3(2xy - 5x)$
- **j** $3(6x - 4y) + x(3 - 2y)$
- **k** $3x^2(4 - x) + 2(5x^2 - 2x^3)$
- **l** $x(x - y) + 3(2x - y)$
- **m** $4(s - 2) + 3s(4 - t)$
- **n** $x(x + y) + x(x - y)$
- **o** $2x(x + y) + 2(x^2 + 3xy)$
- **p** $x(2x + 3) + 3(5 - 2x)$
- **q** $4(2k - 3) + (k - 5)$
- **r** $3(4xy - 2x) + 5(3x - xy)$

## Expanding brackets with negative coefficients

So far the numbers in front of the brackets you expanded were positive. You expand brackets in the same way when there is a negative number before the bracket, but you have to make sure you use the correct signs.

The key is to remember that a '+' or a '−' is attached to the number immediately following it and should be included when you multiply out brackets.

### WORKED EXAMPLE 11

Expand and simplify the following expressions.

**a** $-3(x + 4)$ **b** $4(y - 7) - 5(3y + 5)$ **c** $8(p + 4) - 10(9p - 6)$

**Answers**

**a** $-3(x + 4)$

$-3(x + 4) = -3x - 12$ Remember that the negative sign is attached to the 3.

$-3 \times x = -3x$ and $-3 \times 4 = -12$

**b** $4(y - 7) - 5(3y + 5) = 4y - 28 - 15y - 25$ Remember that both terms in the second bracket are multiplied by −5.

$= -11y - 53$ Collect like terms and simplify.

**TIP**

Remember:

$+ \times + = +$

$+ \times - = -$

$- \times - = +$

WORKED EXAMPLE 11 CONTINUED

| | | |
|---|---|---|
| **c** | $8(p+4) - 10(9p-6) = 8p + 32 - 90p + 60$ | Remove the brackets. Pay attention to the negative signs. |
| | $= -82p + 92$ | Collect like terms and simplify. |

## Exercise 2.7

**1** Expand each of the following and simplify your answers as far as possible.

**a** $-10(3p+6)$ **b** $-3(5x+7)$ **c** $-5(4y+0.2)$

**d** $-3(q-12)$ **e** $-12(2t-7)$ **f** $-1.5(8z-4)$

**g** $-3(2x+5y)$ **h** $-6(4p+5q)$ **i** $-9(3h-6k)$

**j** $-2(5h+5k-8j)$ **k** $-4(2a-3b-6c+4d)$ **l** $-6(x^2+6y^2-2y^3)$

**2** Expand each of the following and simplify your answers as far as possible.

**a** $2-5(x+2)$ **b** $2-5(x-2)$

**c** $14(x-3)-4(x-1)$ **d** $-7(f+3)-3(2f-7)$

**e** $3g-7(7g-7)+2(5g-6)$ **f** $6(3y-5)-2(3y-5)$

**g** $4x(x-4)-10x(3x+6)$ **h** $14x(x+7)-3x(5x+7)$

**i** $x^2-5x(2x-6)$ **j** $5q^2-2q(q-12)-3q^2$

**k** $18pq-12p(5q-7)$ **l** $12m(2n-4)-24n(m-2)$

TIP

Try not to carry out too many steps at once. Show every term of your expansion and then simplify.

**3** Expand each expression and simplify your answers as far as possible.

**a** $8x-2(3-2x)$ **b** $11x-(6-2x)$

**c** $4x+5-3(2x-4)$ **d** $7-2(x-3)+3x$

**e** $15-4(x-2)-3x$ **f** $4x-2(1-3x)-6$

**g** $3(x+5)-4(5-x)$ **h** $x(x-3)-2(x-4)$

**i** $3x(x-2)-(x-2)$ **j** $2x(3+x)-3(x-2)$

**k** $3(x-5)-(3+x)$ **l** $2x(3x+1)-2(3-2x)$

# 2.5 Indices

## Revisiting index notation

When you write a number using indices (powers) you have written it in index notation. Any number can be used as an index including 0, negative integers and fractions. The index tells you how many times the base has been multiplied by itself. So:

| | |
|---|---|
| $3 \times 3 \times 3 \times 3 = 3^4$ | 3 is the base, 4 is the index |
| $a \times a \times a \times a \times a = a^5$ | $a$ is the base, 5 is the index |

## Raising a power

Look at these two examples:

$(x^3)^2 = x^3 \times x^3 = x^{3+3} = x^6 \qquad (2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3 = 2^4 \times x^{3+3+3+3} = 16x^{12}$

By writing in expanded form like this you can see that $(x^3)^2 = x^6$ and $(2x^3)^4 = 16x^{12}$

**When you have to raise a power to another power you multiply the indices: $(x^m)^n = x^{mn}$**

### WORKED EXAMPLE 15

Simplify.

**a** $(x^3)^6$ **b** $(3x^4y^3)^2$ **c** $(p^3)^4 \div (p^6)^2$

**Answers**

**a** $(x^3)^6 = x^{3\times6}$

$= x^{18}$

Multiply the indices.

**b** $(3x^4y^3)^2$

$= 3^2 \times x^{4\times2} \times y^{3\times2}$

$= 9x^8y^6$

Square each of the terms to remove the brackets and multiply the indices.

**c** $(p^3)^4 \div (p^6)^2$

$= p^{3\times4} \div p^{6\times2}$

$= p^{12} \div p^{12}$

$= p^{12-12}$

$= p^0$

$= 1$

Expand the brackets first by multiplying the indices. Divide by subtracting the indices.

**TIP**

A common error is to forget to take powers of the numerical terms. For example, in part (b) you need to square the '3' to give '9'.

## Exercise 2.8

**1** Simplify.

**a** $x^2 \times x^6$ **b** $a^2 \times a^8$ **c** $y^2 \times y^0$ **d** $x^9 \times x^4$

**e** $y^2 \times y^7$ **f** $y^3 \times y^4$ **g** $y \times y^5$ **h** $t \times t^4$

**i** $3x^4 \times 2x^3$ **j** $3y^2 \times 3y^4$ **k** $2m \times m^3$ **l** $3s^3 \times 2s^4$

**m** $5x^3 \times 3$ **n** $8x^4 \times x^3$ **o** $4z^6 \times 2z$ **p** $x^2 \times 4x^5$

**2** Simplify.

**a** $x^6 \div x^4$ **b** $g^{12} \div g^3$ **c** $y^4 \div y^3$ **d** $k^3 \div k$

**e** $\frac{s^5}{s}$ **f** $\frac{x^6}{x^4}$ **g** $\frac{6x^5}{2x^3}$ **h** $\frac{9p^7}{3p^4}$

**i** $\frac{12y^2}{3y}$ **j** $\frac{3x^4}{6x^3}$ **k** $\frac{15x^3}{5x^3}$ **l** $\frac{9b^4}{3b^3}$

**m** $\frac{3x^3}{9x^4}$ **n** $\frac{16a^2b^2}{4ab}$ **o** $\frac{12xy^2}{12xy^2}$

**3** Simplify.

**a** $(a^2)^2$ **b** $(v^2)^3$ **c** $(f^2)^6$ **d** $(y^3)^2$

**e** $(2x^2)^5$ **f** $(3c^2d^2)^2$ **g** $(x^4)^0$ **h** $(5x^2)^3$

**i** $(a^2b^2)^3$ **j** $(x^2y^4)^5$ **k** $(xy^4)^3$ **l** $(4gh^2)^2$

**m** $(3x^2)^4$ **n** $(xy^6)^4$ **o** $\left(\frac{x^2}{y}\right)^0$

**4** Use the appropriate laws of indices to simplify these expressions.

**a** $2x^2 \times 3x^3 \times 2x$ **b** $4 \times 2x \times 3x^2y$ **c** $4k \times k \times k^2$

**d** $(x^2)^2 \div 4x^2$ **e** $11x^3 \times 4(a^2b)^2$ **f** $4x(x^2 + 7)$

**g** $x^2(4x - x^3)$ **h** $x^8 \div (x^3)^2$ **i** $7x^2y^2 \div (x^3y)^2$

**j** $\frac{(4x^2 \times 3x^4)}{6x^4}$ **k** $\left(\frac{a^4}{b^2}\right)^3$ **l** $\frac{x^8 \times (xy^2)^4}{(2x^2)^4}$

**m** $(8x^2)^0$ **n** $4x^2 \times 2x^3 \div (2x)^0$ **o** $\frac{(4x^2y^3)^2}{(2xy)^3}$

**TIP**

When there is a mixture of numbers and letters, deal with the numbers first and then apply the laws of indices to the letters in alphabetical order.

## Negative indices

In Chapter 1 you learned how to use negative numbers as indices. You will now apply those rules to expressions containing letters.

Look at these two methods of working out.

**Using expanded notation:**

$$x^3 \div x^5 = \frac{x \times x \times x}{x \times x \times x \times x \times x}$$

$$= \frac{1}{x \times x}$$

$$= \frac{1}{x^2}$$

**Using the law of indices for division:**

$$x^3 \div x^5 = x^{3-5}$$

$$= x^{-2}$$

This shows that $\frac{1}{x^2} = x^{-2}$.

So, $x^{-m} = \frac{1}{x^m}$ (when $x \neq 0$)

**LINK**

Negative indices are often used in units in physics. For example, you often write 'kilometres per hour' as 'km h⁻¹'.

**TIP**

In everyday language you can say that when a number is written with a negative power, it is equal to '1 over' the number to the same positive power. Another way of saying '1 over' is reciprocal, so $a^{-2}$ can be.written as the reciprocal of $a^2$, i.e. $\frac{1}{a^2}$.

When an expression contains negative indices, you apply the same laws as for other indices to simplify it.

**MATHEMATICAL CONNECTIONS**

Both positive and negative indices are used in standard form. You will learn to use standard form to write very large or very small numbers in Chapter 5.

## WORKED EXAMPLE 16

**1** Write these with a positive index.

**a** $x^{-4}$ **b** $y^{-3}$

**Answers**

**a** $x^{-4} = \frac{1}{x^4}$ **b** $y^{-3} = \frac{1}{y^3}$

**2** Simplify. Give your answers with positive indices.

**a** $\frac{4x^2}{2x^4}$ **b** $2x^{-2} \times 3x^{-4}$ **c** $(3y^2)^{-3}$

**Answers**

**a**
$$\frac{4x^2}{2x^4} = \frac{4}{2} \times x^{2-4} = 2x^{-2} = \frac{2}{x^2}$$

**b**
$$2x^{-2} \times 3x^{-4} = \frac{2}{x^2} \times \frac{3}{x^4} = \frac{6}{x^{2+4}} = \frac{6}{x^6}$$

**c**
$$(3y^2)^{-3} = \frac{1}{(3y^2)^3} = \frac{1}{3^3 \times y^{2\times3}} = \frac{1}{27y^6}$$

The laws of indices can also help you find the value of an index in simple equations. For the same base, if $a^x = a^n$, then $x = n$.

For example, $2^x = 8$. You know that $2^3 = 8$, so $2^x = 2^3$ and $x = 3$.

## WORKED EXAMPLE 17

If $2^x = 128$ find the value of $x$.

**Answer**

$2^x = 128$ Rewrite 128 as a power of 2. You might need to use trial and improvement to do this.

$2^7 = 128$

$\therefore x = 7$

# Exercise 2.9

**1** State whether the following are true or false.

**a** $4^{-2} = \frac{1}{16}$ **b** $8^{-2} = \frac{1}{16}$ **c** $x^{-3} = \frac{1}{3x}$ **d** $2x^{-2} = \frac{1}{x}$

**2** Write each expression so it has only positive indices.

**a** $x^{-2}$ **b** $y^{-3}$ **c** $(xy)^{-2}$ **d** $2x^{-2}$

**e** $12x^{-3}$ **f** $7y^{-3}$ **g** $8xy^{-3}$ **h** $12x^{-3}y^{-4}$

**3** Simplify. Write your answer using only positive indices.

**a** $b^{-3} \times b^4$ **b** $2x^{-3} \times 3x^{-3}$ **c** $4s^3 \div 12s^7$

**d** $\frac{h^{-7}}{h^4}$ **e** $(2x^2)^{-3}$ **f** $(c^{-2})^3$

**g** $x^{-3} \div x^{-4}$ **h** $\frac{x^{-2}}{x^3}$ **i** $a^4b^{-3} \times a^3b^{-2}$

**j** $(x^4y^{-2})^3 \times (xy^3)^{-2}$ **k** $\frac{2x^5y^3}{x^2y^{-4}} \times \frac{y^4}{x^7}$ **l** $\frac{m^3n^{-6}}{m^{-4}n^7} \div \frac{m^5n^{-9}}{m^{-2}n^4}$

**m** $\left(\frac{3m^4n^3}{2mn}\right)^2 \div \frac{3mn}{(2m^{-2}n^3)^4}$

**4** Find the value of $x$ in each equation.

**a** $3^x = 81$ **b** $2^x = 32$ **c** $4^{x-2} = 1$

**d** $5^x = \frac{1}{125}$ **e** $10^{1-x} = \frac{1}{100}$ **f** $2^x + 1 = 9$

**g** $4 \times 3^x = 36$ **h** $3 \times 3^x = 243$

## Summary of index laws

| | |
|---|---|
| $x^m \times x^n = x^{m+n}$ | When multiplying terms, add the indices. |
| $x^m \div x^n = x^{m-n}$ | When dividing, subtract the indices. |
| $(x^m)^n = x^{mn}$ | When finding the power of a power, multiply the indices. |
| $x^0 = 1$ | Any value to the power 0 is equal to 1. |
| $x^{-m} = \frac{1}{x^m}$ | (when $x \neq 0$) |

## Fractional indices

The laws of indices also apply when the index is a fraction. Look at these examples carefully to remind yourself what fractional indices mean in algebra:

$x^{\frac{1}{2}} \times x^{\frac{1}{2}}$

$= x^{\frac{1}{2}+\frac{1}{2}}$ Use the law of indices and add the powers.

$= x^1$

$= x$

In order to understand what $x^{\frac{1}{2}}$ means, ask yourself: what number multiplied by itself will give $x$?

$\sqrt{x} \times \sqrt{x} = x$

So, $x^{\frac{1}{2}} = \sqrt{x}$

$y^{\frac{1}{3}} \times y^{\frac{1}{3}} \times y^{\frac{1}{3}}$

$= y^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$ Use the law of indices and add the powers.

$= y^1$

$= y$

### WORKED EXAMPLE 19

Simplify $(\sqrt[5]{a^2})^{\frac{3}{2}} \times (\sqrt[3]{a^5})^{\frac{1}{5}}$

**Answer**

$(\sqrt[5]{a^2})^{\frac{3}{2}} \times (\sqrt[3]{a^5})^{\frac{1}{5}} = (a^{\frac{2}{5}})^{\frac{3}{2}} \times (a^{\frac{5}{3}})^{\frac{1}{5}}$ Apply the rule $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$= a^{\frac{3}{5}} \times a^{\frac{1}{3}}$ Raise the powers. Simplify the fractions.

$= a^{\frac{3}{5}+\frac{1}{3}}$ Apply laws of indices.

$= a^{\frac{9}{15}+\frac{5}{15}}$ Form equivalent fractions and add them.

$= a^{\frac{14}{15}}$

In Exercise 2.9 you worked out the value of $x$ when it was the exponent in an equation. An equation that requires you to find the exponent is called an exponential equation.

### WORKED EXAMPLE 20

If $2^{x+3} = \frac{1}{16}$ find the value of x.

**Answer**

$2^{x+3} = \frac{1}{16}$ Rewrite the fraction as a power of 2 with a negative index.

$2^{x+3} = 2^{-4}$ Equate the indices.

$x + 3 = -4$

$x = -7$

## Exercise 2.10

1 Simplify.

a $x^{\frac{1}{3}} \times x^{\frac{1}{3}}$ b $x^{\frac{1}{2}} \times x^{\frac{2}{3}}$ c $\left(\frac{y^4}{y^{10}}\right)^{\frac{1}{2}}$ d $\left(\frac{x^6}{y^2}\right)^{\frac{1}{2}}$

e $\frac{a^{\frac{6}{7}}}{a^{\frac{2}{7}}}$ f $\frac{7}{8}b^{\frac{1}{2}} \div \frac{1}{2}b^{-\frac{3}{2}}$ g $\frac{2x^{\frac{2}{3}}}{x^{\frac{8}{3}}}$ h $\frac{9k^{\frac{1}{3}}}{12k^{\frac{4}{3}}}$

i $3(\sqrt[4]{x^7})$ j $\frac{1}{2}x^{\frac{1}{2}} \div 2x^2$ k $-\frac{1}{2}s^{\frac{3}{4}} \div -2s^{-\frac{1}{4}}$ l $\frac{3}{4}x^{\frac{1}{2}} \div \frac{1}{2}x^{-\frac{1}{4}}$

m $-\frac{1}{4}x^{\frac{3}{4}} \div -2x^{-\frac{1}{4}}$ n $\frac{1}{2}x^{\frac{1}{2}} \div 2x^2$ o $\sqrt[3]{x} \times \sqrt[4]{x^3}$ p $\frac{\sqrt[3]{x^2y}}{\sqrt{xy^3}}$

2 Find the value of $x$ in each of these equations.

a $2^x = 64$ b $196^x = 14$ c $x^{\frac{1}{3}} = 7$ d $(x-1)^{\frac{3}{4}} = 64$

e $3^x = 81$ f $4^x = 256$ g $2^{-x} = \frac{1}{64}$ h $3^{x-1} = 81$

i $9^{-x} = \frac{1}{81}$ j $3^{-x} = 81$ k $64^x = 2$ l $16^x = 8$

**TIP**

Remember, simplify means to write in its simplest form. So, if you were to simplify $x^{\frac{1}{5}} \times x^{-\frac{1}{2}}$ you would write:

$= x^{\frac{1}{5}-\frac{1}{2}}$

$= x^{\frac{2}{10}-\frac{5}{10}}$

$= x^{-\frac{3}{10}}$

$= \frac{1}{x^{\frac{3}{10}}}$

## SUMMARY CONTINUED

| Are you able to …? |
|---|
| use letters to represent numbers |
| write expressions to represent mathematical information |
| substitute letters with numbers to find the value of an expression |
| add and subtract like terms to simplify expressions |
| multiply and divide to simplify expressions |
| expand expressions by removing brackets and getting rid of other grouping symbols |
| use and make sense of positive, negative and zero indices |
| apply the laws of indices to simplify expressions |
| work with fractional indices |
| solve exponential equations using fractional indices. |

## Practice questions

1 For a number, $n$, write an expression for:

a the sum of the number and 12 [1]

b twice the number minus four [1]

c the number multiplied by $x$ and then squared [1]

d the square of the number cubed. [1]

2 If $n$ is any positive integer,

a Write an expression that is an even number for all possible values of $n$. [1]

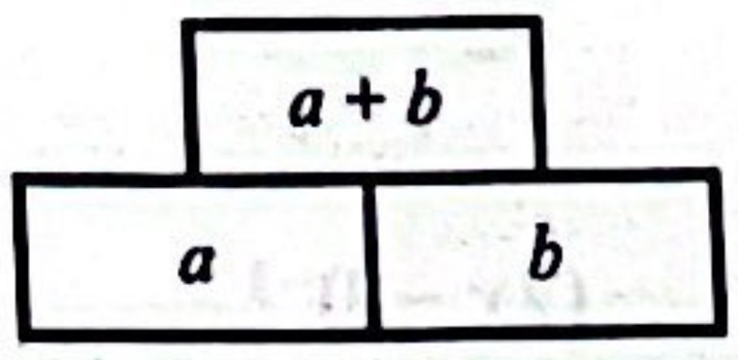
b Explain why $2n + 1$ is always an odd number. [1]

Every positive odd number $p$ can be written in the form $p = 2n + 1$.

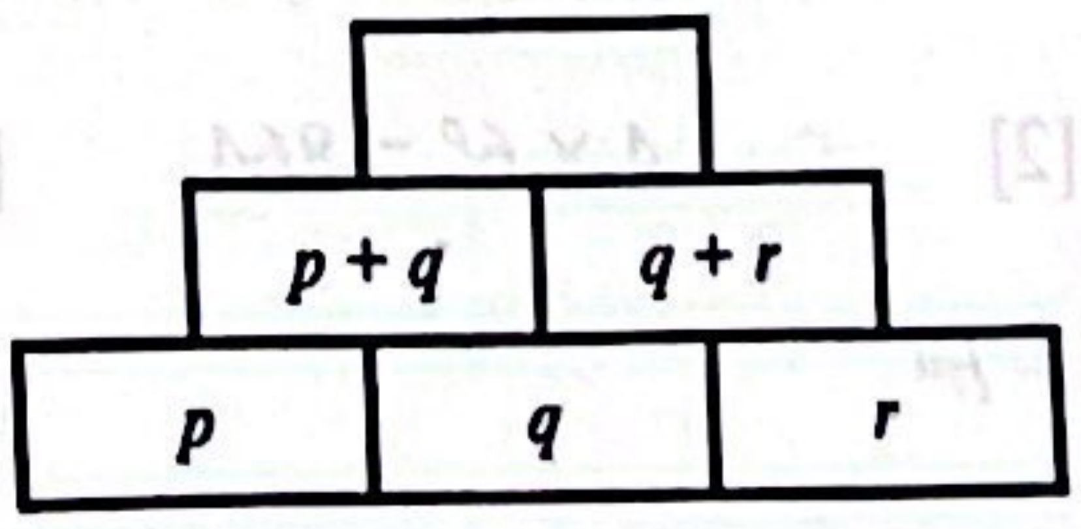
c Write an expression, in terms of $n$, for the next largest odd number after $p$. [1]

d Use your answer to part (c) to show that any two consecutive odd numbers always add up to an even number. [3]

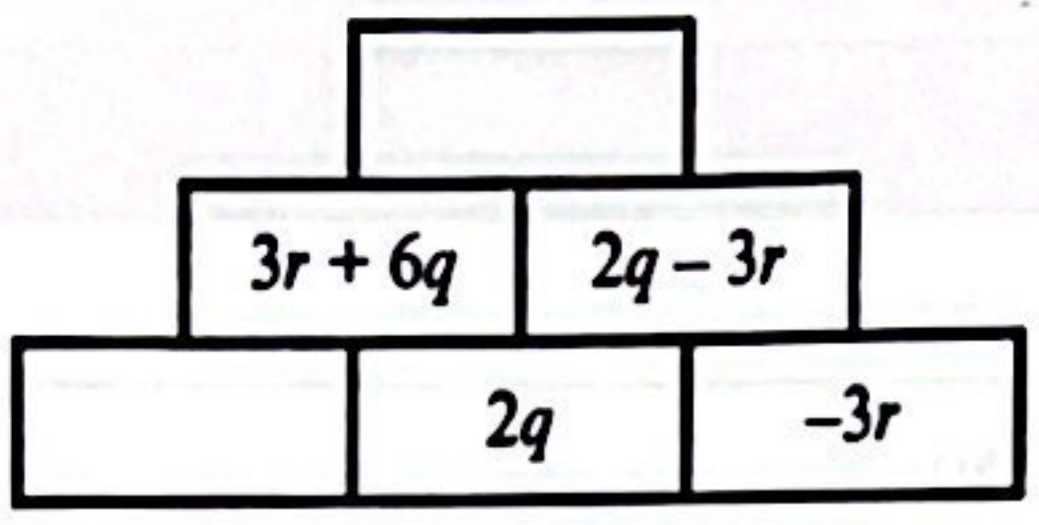
3 Walls are made from bricks with algebraic expressions written on the sides. Each expression is made by adding the two expressions underneath, like this.



a Here is another wall. Write an expression for the brick at the top. [2]



b Make a copy of the next wall and fill in the missing expressions. [3]



c Another wall is made with six bricks as before. The expressions in the bottom bricks are $2h$, $j$, and $2k$ reading left to right, where $h$, $j$ and $k$ are integers. Explain why the top brick always contains an even number. [4]

4 Simplify.

a $9xy + 3x + 6xy - 2x$ [2]   b $6xy - xy + 3y$ [2]

5 Simplify.

a $\dfrac{a^3b^4}{ab^3}$ [2]   b $2(x^3)^2$ [2]   c $3x \times 2x^3y^2$ [2]

d $(4ax^2)^0$ [2]   e $4x^2y \times x^3y^2$ [2]   f $3x^{-4} \times 5x^6$ [2]

g $\dfrac{3x^5}{7x^4} \div \dfrac{6x^{-6}}{14x^{-4}}$ [3]   h $(4x^{-5})^2$ [2]   i $\left(\dfrac{3x}{4y}\right)^3$ [3]

j $\dfrac{4x^{12}y^{-3}}{12x^{-7}y^9}$ [3]   k $\dfrac{14p^5q^{-4}}{30p^4q^4} \times \dfrac{5pq^{-7}}{2p^{-4}q^5}$ [3]

6 Simplify $7x^3y^2 \times (2x)^3 - (4x^3y)^2 - 4xy^2 \times 10x^5$ [3]

7 Find the value of $(x + 5) - (x - 5)$ when:

a $x = 1$ [1]   b $x = 0$ [1]   c $x = 5$ [1]

8 $s = \frac{1}{2}(u + v)t$

Without using a calculator find $s$ if $u = \frac{2}{5}$, $v = 4\frac{1}{2}$, $t = 3$.

Write your answer as a simplified fraction. [3]

9 Expand each expression and simplify if possible.

a $5(x - 2) + 3(x + 2)$ [3]   b $5x(x + 7y) - 2x(2x - y)$ [3]

10 a $m(m - n) - n(n - m)$ [3]

b $x(y - z) + y(z - x) + z(x - y)$ [3]

11 Simplify and write the answers with positive indices only.

a $x^5 \times x^{-2}$ [2]   b $\dfrac{8x^2}{2x^4}$ [2]   c $(2x - 2)^{-3}$ [2]

12 Find the value of each unknown when:

a $4^x = 64$ [2]   b $3^x - 5 = 22$ [2]   c $4 \times 6^p = 864$ [2]

13 If $a = 3$, $b = 2$ and $c = -1$, find the value of $a^b - c^a + b^a$ [2]

**14** Simplify.

**a** $3x^{\frac{1}{2}} \times 5x^{\frac{1}{2}}$ [2] **b** $(81y^6)^{\frac{1}{2}}$ [2] **c** $(64x^3)^{\frac{1}{3}}$ [2]

**15** Find the value of $x$ when:

**a** $\left(\frac{1}{2}\right)^x = 8$ [2] **b** $3^x = \frac{1}{27}$ [2] **c** $125^x = 5$ [2] **d** $125^x = \frac{1}{5}$ [2]

**16** $p = 2^x$ and $q = 2^y$

Find, in terms of $p$ and $q$:

**a** $2^{x+y}$ [2] **b** $2^{x+y-2}$ [2] **c** $2^{3x}$ [2]

**17** Find the value of $n$ for which:

**a** $n^{-1} = 2^{-2}$ [2] **b** $4^n = \left(\sqrt[4]{32}\right)^3$ [2]

## SELF ASSESSMENT

Mark your answers to the practice questions.
Complete these statements in your book.

- I now know …
- I need to know more about …
- These things went well …
- I could do better if I …